Constraints on two-neutron separation energy in the Borromean ^{22}C nucleus

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Abstract

The recently extracted matter radius of carbon isotope ^{22}C allows us to estimate the mean-square distance of a halo neutron with respect to the center-of-mass of this nucleus. By considering this information, we suggest an energy region for an experimental investigation of the unbound ^{21}C virtual state. Our analysis, in a renormalized zero-ranged three-body model, also indicates that the two-neutron separation energy in ^{22}C is expected to be found below ∼0.4 MeV, where the ^{22}C is approximated by a Borromean configuration with a pointlike ^{20}C and two s-wave halo neutrons. A virtual-state energy of ^{21}C close to zero, would make the ^{22}C, within Borromean nuclei configurations, the most promising candidate to present an excited bound Efimov state or a continuum three-body resonance.

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1 Introduction

In the breakthrough experiment reported by K. Tanaka et al. [1], the matter radius of the carbon isotope ^{22}C was extracted via a finite-range Glauber

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analysis under an optical-limit approximation of the reaction cross section, for ^{22}C on a liquid hydrogen target, measured around 40 MeV/nucleon. The extracted matter radius presents a huge value of 5.4±0.9 fm (for a viewpoint on [1], see also Ref. [2].), which characterizes this nucleus as the heaviest halo nuclei discovered until now. For the two-neutron separation energy, S_{2n}, they also quote a value of 0.42±0.94 MeV. These experimental results, together with other well-known properties of carbon isotopes [3,4,5,6], indicate that ^{22}C is weakly-bound, having a very large two-neutron halo with the ^{20}C as a core, such that the corresponding observables are probably dominated by the tail of the three-body wave function in an ideal s-wave three-body model, as considered in Ref. [7]. In addition, within a $n-n-{}^{20}$C configuration, we have the ^{22}C as a Borromean halo system, considering that a neutron (n) and ^{20}C is known as an unbound system.

In view of its very low-energy properties, within the $n-n-$core halo-nuclei systems, the nucleus ^{20}C has been cited previously [3,4,8,9,10] as a good candidate to present three-body Efimov states [11]. Considering that this nucleus is more compact than the ^{22}C, with its ground state in a probable $(0d_{5/2})^6$ configuration, it is also quite natural to suggest the ^{22}C as being still more favorable to have an Efimov state [7], with its halo predominantly produced by a $(1s_{1/2})^2$ component. However, to infer about the possibility of existence of Efimov excited states in an ideal s-wave two-neutron halo nucleus like ^{22}C [7] is crucial to have a measurement of the virtual state energy of ^{21}C.

The characteristics of ^{22}C, roughly described in the first and second paragraphs, allow us to use a Dirac$-\delta$ (zero-range) interaction, as reviewed in Refs. [6,12], acting on s-wave to study this problem. In the zero-range limit three scales emerge for describing the full long-range structure of the $n-n-{}^{20}$C wave function: the virtual $n-n$ energy, the s-wave virtual state energy of the neutron in ^{21}C and the two-neutron separation S_{2n}. The information on the unbound $n-{}^{20}$C virtual energy is unknown and S_{2n} has an uncertainty that is about twice its own value.

In this study, we calculate a region to guide the experiments to search for the ^{22}C two-neutron separation energy. By first considering that the virtual state energy of ^{21}C is varying from 0 to 100 keV, and that the bound-state energy of ^{22}C is given in an interval from 100 to 1500 keV, we calculate the mean-square distance of the halo neutron to the center-of-mass (CM) of the corresponding three-body system as a function of S_{2n}. Then, by using the extracted one-nucleon mean-distance r_n and its uncertainties as constraints, we are able to estimate a reasonable region for the search of the two-neutron separation energy in ^{22}C, as well as the corresponding region of the virtual state energy of ^{21}C (directly related to a negative scattering length of the $n-{}^{20}$C system).

2 Neutron-neutron-^{20}C model

The three-body halo wave function allows us to calculate the neutron mean-square distance to the corresponding three-body CM. This model has already been applied with success to describe halo radii in Ref. [13] and two-neutron correlation functions in Ref. [14].

The available quantity that can be used to define limits on the two-neutron halo ^{22}C binding is the extracted matter radius, $R_M^{^{22}\mathrm{C}} = 5.4 \pm 0.9$ fm, that was recently reported in Ref. [1]. The root-mean-square distance, from the CM of the $n-n-^{20}$C to one of its halo neutrons, can be estimated by using the additional information on the matter radius of the loosely bound ^{20}C, which is given in Ref. [15] ($R_M^{^{20}\mathrm{C}} = 2.98 \pm 0.05$ fm). In view of the large difference between the radius of ^{22}C and ^{20}C, we consider it is a reasonable approximation to assume ^{20}C as the core for the present purpose, such that we still can use a three-body approach. The result of our estimation is given by the following:

$$r_n \simeq \sqrt{\frac{22}{2}\left[(R_M^{^{22}\mathrm{C}})^2 - \left(\frac{20}{22}R_M^{^{20}\mathrm{C}}\right)^2\right]} \approx 15 \pm 4 \text{ fm} , \tag{1}$$

where $r_n \equiv \sqrt{\langle r_n^2 \rangle}$ and $R_M^{^i\mathrm{C}} \equiv \sqrt{\langle (R_M^{^i\mathrm{C}})^2 \rangle}$, with $i = 20, 22$. This simple approximation shows that ^{22}C is the largest known halo along the neutron dripline. By using this value, we will be able to define a region where the ^{21}C virtual energy can be found, as well as the corresponding two-neutron separation energy, S_{2n}, in ^{22}C.

2.1 Subtracted Faddeev Equations

In the following, the Faddeev formalism is developed by considering a renormalized zero-range three-body model for a system with a core (c), which will be the ^{20}C in the present work, and two-identical particles (the neutrons). The mass of the core is given by $m_c = Am_n$, where A defines the mass ratio and m_n is the neutron mass. Throughout this article, we will use units such that $\hbar = m_n = 1$. In the renormalization procedure, the kernel regularization is done via a subtraction method also considered in [13]. After partial wave projection, the s-wave coupled subtracted integral equations, for two neutrons and a core, can be written in momentum space by a coupled equations for the spectator functions $\chi_c(x) \equiv \phi_c(x)/x$ and $\chi_n(x) \equiv \phi_n(x)/x$, as follows:

$$\phi_c(y) = 2\tau_{nn}(y;\epsilon_3) \int_0^\infty dx\ G_1(y,x;\epsilon_3)\phi_n(x), \tag{2}$$

$$\phi_n(y) = \tau_{nc}(y;\epsilon_3) \int_0^\infty dx\ \left[G_1(x,y;\epsilon_3)\phi_c(x) + AG_2(y,x;\epsilon_3)\phi_n(x)\right], \tag{3}$$

where

$$\tau_{nn}(y;\epsilon_3) \equiv \frac{1}{\pi}\left[\sqrt{\epsilon_3 + \frac{A+2}{4A}y^2} + \sqrt{\epsilon_{nn}}\right]^{-1}, \tag{4}$$

$$\tau_{nc}(y;\epsilon_3) \equiv \frac{1}{\pi}\left(\frac{A+1}{2A}\right)^{3/2}\left[\sqrt{\epsilon_3 + \frac{A+2}{2(A+1)}y^2} + \sqrt{\epsilon_{nc}}\right]^{-1}, \tag{5}$$

$$G_1(y,x;\epsilon_3) \equiv \log\frac{2A(\epsilon_3 + x^2 + xy) + y^2(A+1)}{2A(\epsilon_3 + x^2 - xy) + y^2(A+1)} - \log\frac{2A(1 + x^2 + xy) + y^2(A+1)}{2A(1 + x^2 - xy) + y^2(A+1)}, \tag{6}$$

$$G_2(y,x;\epsilon_3) \equiv \log\frac{2(A\epsilon_3 + xy) + (y^2+x^2)(A+1)}{2(A\epsilon_3 - xy) + (y^2+x^2)(A+1)} - \log\frac{2(A + xy) + (y^2+x^2)(A+1)}{2(A - xy) + (y^2+x^2)(A+1)}. \tag{7}$$

In the above, we are using the odd-man-out notation for the spectator functions χ. The indexes n or c in χ indicates the spectator particle. The momentum and energy variables are written in terms of a momentum three-body scale $\mu_{(3)}$, which is used in our subtractive regularization procedure to renormalize the originally singular Faddeev equations. The units considered in Eqs. (2-7) are such that all quantities are dimensionless. In view of that, the corresponding dimensionless energies for the three-body system are given by $\epsilon_3 \equiv S_{2n}/\mu^2_{(3)}$, $\epsilon_{nn} \equiv -E_{nn}/\mu^2_{(3)}$, $\epsilon_{nc} \equiv -E_{nc}/\mu^2_{(3)}$, where $E_{nn} = -143$ keV and E_{nc} are, respectively, the $n-n$ and the $n-{}^{20}$C virtual-state energies.

2.2 The form factor and the mean-square radius

The mean-square distance of the neutron to the CM of the three-body system is calculated from the derivative of the Fourier transform of the respective matter density with respect to the square of the momentum transfer. The Fourier transform of the one-body densities defines the respective form factor, $F_n(q^2)$, as a function of the dimensionless momentum transfer $\vec{q}$. Thus, for the mean-square distance of the neutron to the CM of ^{22}C, we have [13]

$$\langle r_n^2\rangle = -6\left(\frac{21}{22}\right)^2 \left.\frac{dF_n(q^2)}{dq^2}\right|_{q^2=0}, \tag{8}$$

where the form factor is defined as:

$$F_n(q^2) = \int d^3p \; d^3k \; \Psi_n \left(\vec{p} + \frac{\vec{q}}{2}, \vec{k} \right) \Psi_n \left(\vec{p} - \frac{\vec{q}}{2}, \vec{k} \right) \; . \tag{9}$$

The above three-body wave function, Ψ_n, in momentum space are given in terms of the spectator functions χ as:

$$\begin{aligned} \Psi_n(\vec{p}, \vec{k}) = & \left(\frac{1}{S_{2n} + \frac{A+1}{2A}\vec{k}^2 + \frac{A+2}{2(A+1)}\vec{p}^2} - \frac{1}{\mu_3^2 + \frac{A+1}{2A}\vec{k}^2 + \frac{A+2}{2(A+1)}\vec{p}^2} \right) \\ & \times \left[\chi_c \left(\left| \vec{z} - \frac{A\vec{y}}{A+1} \right| \right) + \chi_n \left(|\vec{y}| \right) + \chi_n \left(\left| \vec{z} + \frac{\vec{y}}{A+1} \right| \right) \right] , \end{aligned} \tag{10}$$

where $\vec{k} \equiv \vec{z}\mu_3$ is the relative momentum of the pair and $\vec{p} \equiv \vec{y}\mu_3$ is the relative momentum of the spectator particle to the pair.

3 Results and Conclusion

The calculation of the neutron average distance to the CM of ^{22}C demands as input the S_{2n}, the energies of the virtual s-wave states of the $n-n$ and ^{21}C systems. The unbound ^{21}C virtual state is poorly known. In our model we assumed small values of this virtual state between 0-100 keV. The one-neutron mean distance to the CM, $r_n \equiv \sqrt{\langle r_n^2 \rangle}$, derived from Eqs. (8) and (9) and using the wave function (10) can be written as a general function $\mathcal{R}_n$, dependent on the two-body energies, as:

$$r_n = \mathcal{R}_n \left(\pm\sqrt{\epsilon_{nn}\mu_{(3)}^2}, \pm\sqrt{\epsilon_{nc}\mu_{(3)}^2} \right) \; , \tag{11}$$

where the plus sign (minus) refers to bound (virtual) two-body subsystem. The value of the separation energy is given by $\epsilon_3 = S_{2n}/\mu_{(3)}^2$. To convert all results of the calculations to the physical units we have to introduce the physical value of S_{2n} in (11). In this case the value of the parameters ϵ_{nn} and ϵ_{nc} are determined as:

$$\epsilon_{nn} = -\frac{E_{nn}}{\mu_{(3)}^2} = -\frac{E_{nn}}{S_{2n}}\epsilon_3 \;\text{ and }\; \epsilon_{nc} = -\frac{E_{nc}}{S_{2n}}\epsilon_3 \; . \tag{12}$$

From (11) and (12) , the average distance from the neutron to the CM of the system is given by

$$r_n = \frac{1}{\sqrt{S_{2n}}} \mathcal{R}_n \left(-\sqrt{\frac{|E_{nn}|}{S_{2n}}\epsilon_3}, -\sqrt{\frac{|E_{nc}|}{S_{2n}}\epsilon_3} \right) \ . \tag{13}$$

The limit cycle [16] is achieved when ϵ_{nn} and ϵ_{nc} tends to zero and it is used to compute the radius of the shallowest $n - n - c$ bound state. Therefore, in this limit, the dependence on ϵ_3 can be dropped out:

$$r_n = \frac{1}{\sqrt{S_{2n}}} \mathcal{R}_n \left(-\sqrt{\frac{|E_{nn}|}{S_{2n}}}, -\sqrt{\frac{|E_{nc}|}{S_{2n}}} \right) \ . \tag{14}$$

In practice such limit is achieved fast and the first cycle is enough for the application we are considering (see Ref. [17]).

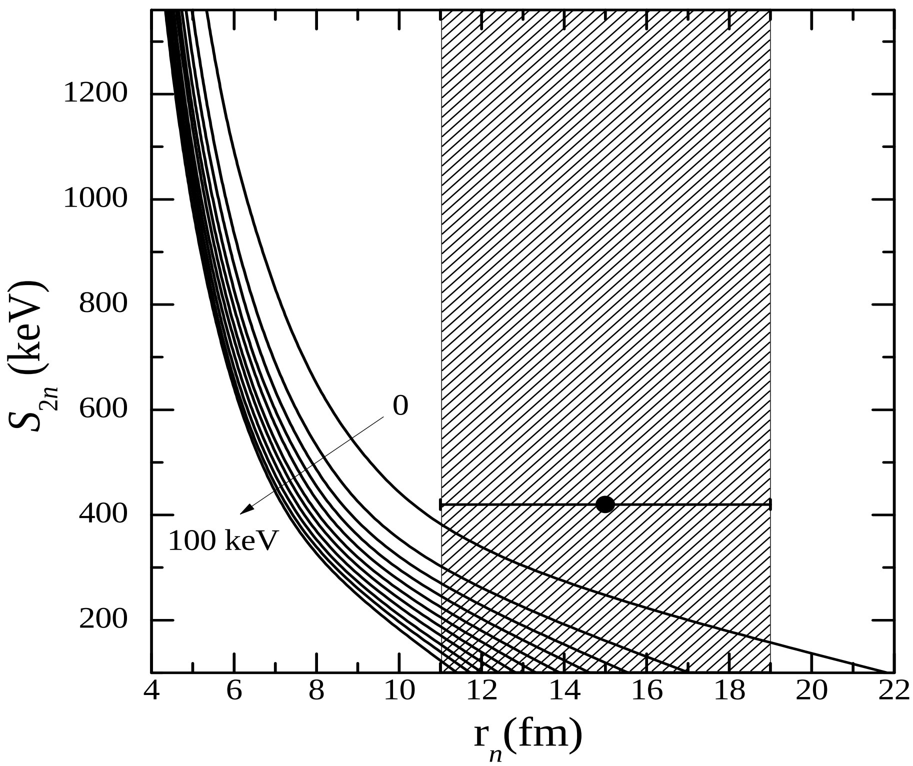

Fig. 1. Two halo neutron separation energies in ^{22}C (S_{2n}) are given in terms of root-mean-square distances of a halo neutron with respect to the three-body CM (r_n). Each curve is calculated for a given ^{21}C virtual-state energy, varying in steps of 10 keV, from 0 to 100 keV (indicated by the arrow). The shaded area, involving the experimental point, corresponds to the region defined by 100 keV $\leq S_{2n} \leq$ 1360 keV, with 11 fm $\leq r_n \leq$ 19 fm.

From experimental data, we have $r_n = 15 \pm 4$ fm, as given by Eq. (1), and the singlet $n - n$ virtual state energy $E_{nn} = -143$ keV. Therefore, in order to use the model results from (14), we have to assume values for the unknown virtual state energy of ^{21}C, to be able to get some information on S_{2n} in ^{22}C. In Fig.1, we display our results for the separation energy for different values

of the s-wave neutron virtual state in ${}^{21}C$, ranging from 0 up to 100 keV. The experimental values of $S_{2n}^{(exp)} = 0.42 \pm 0.94$ MeV [1] and $r_n = 15 \pm 4$ fm are shown in the figure.

We observe that, for a given S_{2n}, the r_n decreases as the absolute value of the virtual state energy increases. This can be explained as follows: as the virtual state energy increases, the interaction between the neutron and the core becomes weaker. Therefore, one can fix a given three-body energy by decreasing the size of the system [13]. By taking into account the value of 15 ± 4 fm, one obtains S_{2n} below $\sim$0.4 MeV for a neutron in ^{21}C bound at the threshold. This result is not far from the central experimental value of 0.42 MeV. We note that a small increase in the virtual state energy up to 20 keV, drops the upper limit of S_{2n} to $\sim$0.3 MeV. Indeed, the finite-range Glauber analysis under an optical-limit approximation of the reaction cross section, for ^{22}C on a liquid hydrogen target, measured around 40 MeV/nucleon [1], indicates that the observed large enhancement of the cross-section compared to the neighbor carbon isotopes, suggests that values of S_{2n} below 0.4 MeV would be possible.

The three-body approximation that we have considered for ^{22}C, where the ^{20}C is the core, can be justified by comparing the size of ^{20}C with the mean distance of the halo neutrons of ^{22}C and also considering that the halo neutrons in ^{20}C are bound with about 3.5 MeV, one order of magnitude greater than S_{2n} in ^{22}C. Thus, the halo neutrons in ^{22}C have a much larger probability to experience the long-range $1/r^2$ potential derived by Efimov than in ^{20}C, as the corresponding wave function tail is extending far beyond the size of ^{20}C. Therefore, the Efimov physics should be much more evident in the properties of ^{22}C ground state than in the corresponding properties of ^{20}C.

In a microscopic 5-body description, beyond the present model, the four neutrons out of ^{18}C, should be in a fully antisymmetric wave function due to the proposed separation of scales. As the s-wave radial wave functions corresponding to the neutrons in the halo of ^{20}C and in ^{22}C have different sizes, an antisymmetric wave function can be built. If all spectator neutron interactions are dominated by only s-waves, as in our model, the Pauli exclusion principle would make the halo neutrons in ^{22}C much less bound than in ^{20}C, which indeed seems to be the case. In essence, with the above remarks, we should emphasize that our three-body model for ^{22}C is not excluding a three-body model for ^{20}C as having a two-neutron halo or an Efimov state for ^{20}C very near the scattering threshold [4].

One possible correction to our results is due to the interaction range. Range corrections in the calculation of different mean distances were performed by Canham and Hammer [19]. By taking the ^{11}Li (Borromean $n-n-{}^{9}$Li system) for comparison, where $S_{2n} \sim 300$ keV and the neutron average distance to the CM is around 6 fm, the correction is a fraction of 1 fm [19] for a fixed

S_{2n}, ^{10}Li virtual state energy and nn scattering length. Therefore, we also expect corrections of the same magnitude, or even smaller, considering that the core is larger but the average distance of the neutron to the CM is more than twice. We should stress that effects from the detailed core dynamics in our calculation are implicitly carried out by the three-body energy, which in our framework is an external parameter.

Summarizing, from the extracted matter radius of ^{22}C, by using a renormalized three-body zero-range model, we estimate the mean-square distance of a halo neutron with respect to the CM of the ^{22}C. From such estimate, we suggest a possible region for an experimental search of both S_{2n} of ^{22}C and the $n-^{20}$C virtual state energy. The ^{22}C is approximated by a Borromean three-body system composed by a point-like core of ^{20}C and two s-wave halo neutrons. The validity of our findings relates to the assumption of a large halo compared to the typical range of the nuclear interaction. We are confident that the guidance provided by this work would help the search for the ^{22}C energy from an experimental analysis.

Finally, based on Fig. 1 of Ref. [4], where it is calculated a region for the appearance of excited Efimov states, we would like to mention that a ^{21}C with an energy close to zero can make the ^{22}C as the most promising Borromean candidate to present excited Efimov states, or a continuum resonance [18].

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